

## MATHEMATICS

9709/02
Paper 2 Pure Mathematics 2 (P2)
October/November 2007
1 hour 15 minutes
Additional Materials: Answer Booklet/Paper Graph Paper List of Formulae (MF9)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet. Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 Show that

$$
\begin{equation*}
\int_{1}^{4} \frac{1}{2 x+1} \mathrm{~d} x=\frac{1}{2} \ln 3 \tag{4}
\end{equation*}
$$

2 The sequence of values given by the iterative formula

$$
x_{n+1}=\frac{2 x_{n}}{3}+\frac{4}{x_{n}^{2}}
$$

with initial value $x_{1}=2$, converges to $\alpha$.
(i) Use this iterative formula to determine $\alpha$ correct to 2 decimal places, giving the result of each iteration to 4 decimal places.
(ii) State an equation that is satisfied by $\alpha$ and hence find the exact value of $\alpha$.

3 (i) Solve the inequality $|y-5|<1$.
(ii) Hence solve the inequality $\left|3^{x}-5\right|<1$, giving 3 significant figures in your answer.

4 The equation of a curve is $y=2 x-\tan x$, where $x$ is in radians. Find the coordinates of the stationary points of the curve for which $-\frac{1}{2} \pi<x<\frac{1}{2} \pi$.

5 The polynomial $3 x^{3}+8 x^{2}+a x-2$, where $a$ is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(x+2)$ is a factor of $\mathrm{p}(x)$.
(i) Find the value of $a$.
(ii) When $a$ has this value, solve the equation $\mathrm{p}(x)=0$.

6 (i) Express $8 \sin \theta-15 \cos \theta$ in the form $R \sin (\theta-\alpha)$, where $R>0$ and $0^{\circ}<\alpha<90^{\circ}$, giving the exact value of $R$ and the value of $\alpha$ correct to 2 decimal places.
(ii) Hence solve the equation

$$
\begin{equation*}
8 \sin \theta-15 \cos \theta=14 \tag{4}
\end{equation*}
$$

giving all solutions in the interval $0^{\circ} \leqslant \theta \leqslant 360^{\circ}$.

7 (i) Prove the identity

$$
\begin{equation*}
(\cos x+3 \sin x)^{2} \equiv 5-4 \cos 2 x+3 \sin 2 x \tag{4}
\end{equation*}
$$

(ii) Using the identity, or otherwise, find the exact value of

$$
\begin{equation*}
\int_{0}^{\frac{1}{4} \pi}(\cos x+3 \sin x)^{2} d x \tag{4}
\end{equation*}
$$

8


The diagram shows the curve $y=x^{2} \mathrm{e}^{-x}$ and its maximum point $M$.
(i) Find the $x$-coordinate of $M$.
(ii) Show that the tangent to the curve at the point where $x=1$ passes through the origin.
(iii) Use the trapezium rule, with two intervals, to estimate the value of

$$
\begin{equation*}
\int_{1}^{3} x^{2} \mathrm{e}^{-x} \mathrm{~d} x \tag{3}
\end{equation*}
$$

giving your answer correct to 2 decimal places.

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